Thermal Resistance of an Eddy

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By experimental means a relation is obtained between the thermal resistance of an eddy and its angular momentum. The eddy is stationary, and no extraneous motion is present. The secondary motion which may develop in the annulus between concentric rotating cylinders is used to obtain the eddies. The fluid motion is well defined at all times and at all points of space. Heat is passed through the eddies, and the Nusselt number is obtained, which varies linearly with the angular momentum. Both Nusselt number and angular momentum vary linearly with the peripheral velocity of the inner rotating cylinder, which can be interpreted in terms of a Reynolds number associated with fluid flow perpendicular to a cylinder.

In the field of heat transfer one often speaks of eddies or secondary fluid motion and of the effect of such motion on the heat transfer coefficient of a system. There are many data which show the over-all effect of secondary eddies, but such motion is usually accompanied by other types of flow. There is relatively little, if any, published information bearing on the heat transfer coefficient of a single eddy. It was decided therefore to study the heat transfer characteristics of an eddy of known motion and to compare the heat transferred by the eddy owing to its angular momentum.

The first problem is to obtain an eddy which is free from extraneous motion. This is done by utilizing the secondary motion formed when a fluid is contained in the annulus of two concentric cylinders and the inner one is rotated above some critical speed. Figure 1 is a schematic drawing of the system in which the curved arrow indicates the secondary motion.

The critical angular velocity of the inner cylinder above which the secondary motion appeared was shown by Taylor (*1*) to be

$$\omega_{cr}^{\ 2} = \frac{\nu^2\pi^4(R_1 + R_2)}{2P(R_2 - R_1)^3 R_1^{\ 2}}$$

where P = constant depending on the geometry.

At cylinder speeds above this value, fluid particles no longer travel in concentric circles about the axis of rotation but travel in spirals while progressing around the annulus. Figure 2 is a photograph of this secondary motion where the cross-sectional picture is taken as indicated in Figure 3. The secondary motion is of an eddy type where the eddies occur in counterrotating pairs along the annulus. In the system that is presented the length and thickness of the annulus are chosen so that the eddy cells are square. The eddies so formed are stable at speeds substantially above the critical speed. As the speed of the inner cylinder increases, the secondary velocities in the individual eddies increase. It was shown by Hagerty (*2*) and later verified in this work that the general shape of the eddies persists for an appreciable range of higher speeds. The fact that the general form does not change materially simplifies computation of the angular momentum.

In the resulting motion the paths of the fluid particles are clearly defined at all points of the space; therefore no distinction is necessary between the region near the boundary (boundary layer) and the region near the center of the annulus. The heat transfer characteristics of the entire region are compared with the angular momentum of the fluid about an axis tangential to the cylinders.

ANGULAR MOMENTUM

The first step is to measure the angular momentum of the eddies. This is done by the use of motion pictures, as shown in Figure 3, which illustrates the method by which the pictures were taken for different speeds of the inner cylinder. The angular momentum in this case is obtained from the motion relative to an axis perpendicular to the plane of the eddy. This method is equivalent to assuming a two-dimensional or plane motion, which is best approximated if the annulus thickness is small compared with the cylinder radius. Taylor (*1*) showed that at instability the form of the eddies can be represented by the stream function

$$\Psi = f(x) \cos \frac{\pi y}{d} \qquad (1)$$

where y is the coordinate measured parallel to the axis of rotation of the inner cylinder and x is measured in the radial direction (Figure 4).

Then

$$u = -\frac{\partial \Psi}{\partial y}; \qquad v = \frac{\partial \Psi}{\partial x} \qquad (2)$$

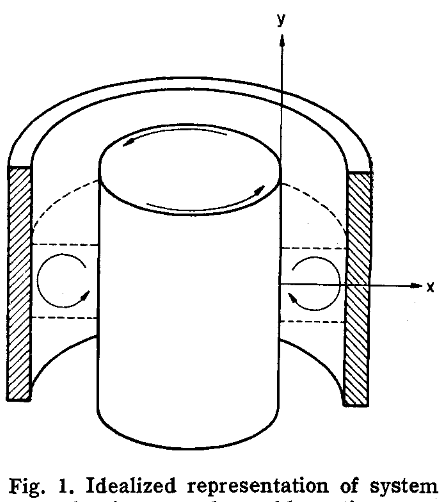


Fig. 1. Idealized representation of system showing secondary eddy motion.

The velocities u and v and the coordinates x and y are defined in Figure 4. If the eddies retain this general shape at speeds above the critical one needs only to determine v at $y = 0$ experimentally to obtain a complete equation for Ψ. If the x axis passes through the center of the eddy, then $v_{y=0} = (\partial\Psi/\partial x)_{y=0} = \partial f/\partial x$ and $u_{y=0} = 0$ and so v need be measured only along the x axis. It was found that the eddies did retain their general form for the range of speeds considered.

Motion pictures are taken at four different speeds above the critical. The values of $v_{y=0}$ are obtained by projecting the pictures on a grid and studying the motion of entrained aluminum filings, frame by frame. The velocity curves are obtained for speeds of 20.0, 26.1, 33.7, and 46.3 rev./min. Figure 5 shows the resulting velocity profile for a speed of 20.0 rev./min.

The values of v from Figure 5 do not satisfy continuity for plane motion, and to allow for the curvature of the cylinders a small correction must be made (see Figure 6) by multiplying each curve by

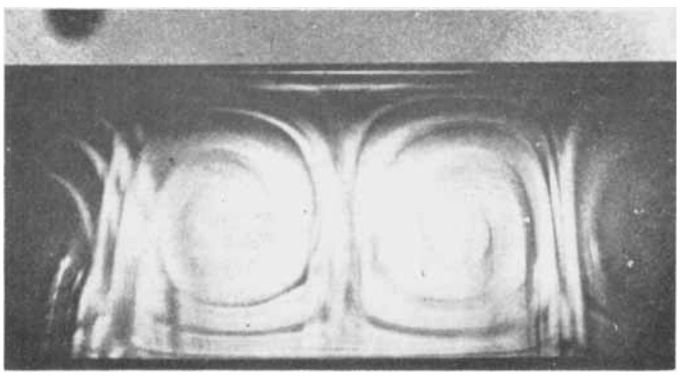


Fig. 2. Cross section of eddies formed between concentric rotating cylinders.

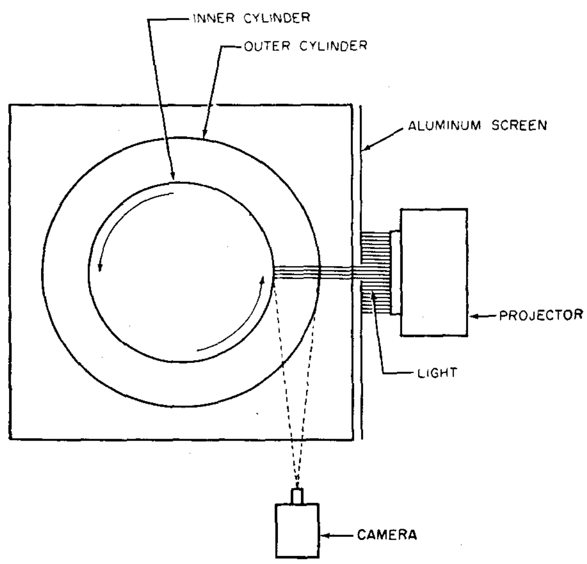


Fig. 3. Arrangement for photographing motion within eddies.

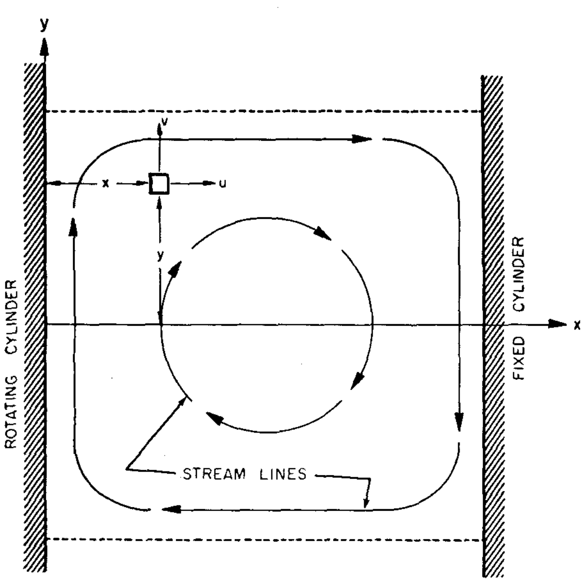
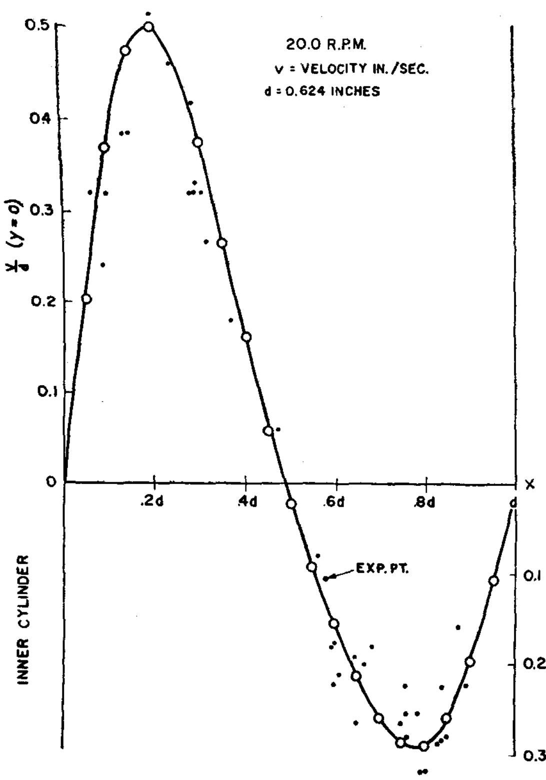


Fig. 4. Coordinates used to describe motion of the eddy.



$$\frac{V}{d} = 3.2(.0152 \sin \pi X/d + .117 \sin 2\pi X/d + .0245 \sin 3\pi X/d + .0210 \sin 4\pi X/d + .00224 \sin 5\pi X/d)$$

Fig. 5. Typical velocity profile across the section of an eddy.

$[1 + (x_1/R_1)]$ where R_1 is the radius of the inner cylinder.

The resulting corrected velocity curves for the four cylinder speeds are shown in Figure 7. It should be noted that the mass could have been corrected in this way instead of the velocity without producing any effect on the angular momentum. The velocity was changed so as to give better visual comparison with a plane eddy.

The experimental-data curves are now approximated by a harmonic analysis, and values of u and v are obtained. As an example the velocities at a speed of 20.0 rev./min. can be written as

$$u = d \sin (\pi y/d) [0.2286 - 0.211 \cos (2\pi x)/d - 0.018 \cos (4\pi x)/d + 0.0004 \cos (6\pi x)/d]$$
$$v = d \cos (\pi y)/d [0.44 \sin (2\pi x)/d + 0.072 \sin (4\pi x)/d - 0.0024 \sin (6\pi x)/d] \quad (3)$$

The angular momentum per foot of eddy is written as (see Figure 4) angular momentum

$$= \int_R (uy - vx)\rho \, dxdy \quad (4)$$

where R is the region of one eddy one unit long.

By use of the values from Equations (3), Equation (4) can be written in the form angular momentum $= 2(\rho d^4)/(\pi^2)H$ (1 unit of eddy) (5)

where H is a number depending on the velocity distribution in the eddy and has the unit of time. When the velocities at all speeds are used in Equation (4), Figure 8 then shows the plotted values of the angular-momentum number against n, the speed of the inner cylinder in revolutions per minute. The value of H at $n = 46.3$ may not be accurate owing to difficulties in measurement at this speed. The curve of Figure 8 gives slightly high values because of dynamic dissimilarity between the optical system and the thermal system that is to be used. The proper correction gives the lower dotted line. This correction is based on the assumption that the rate of increase of angular momentum with cylinder speed is the same for a critical speed of 17.5 rev./min. as it is for a critical speed of 14 rev./min. The curve shows the linear relationship between the angular momentum and the cylinder speed for speeds not too far above critical. It is assumed that the heat transfer coefficient varies linearly with the revolutions per minute of the rotating cylinder. If there is an analogy between heat transfer and momentum transfer and if the rate of momentum transfer in this case can be expressed as an angular momentum, then the heat transfer coefficient of the eddies, h_e, and H should vary in the same manner with a change in revolutions per minute.

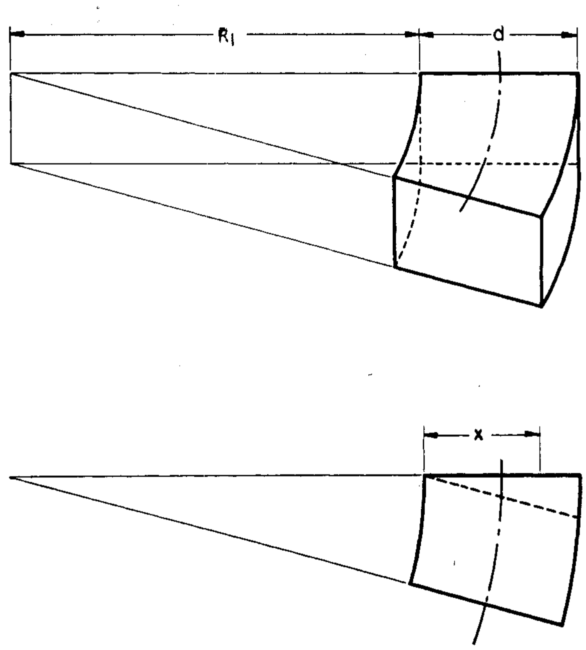


Fig. 6. Effect of curvature on geometry of the system.

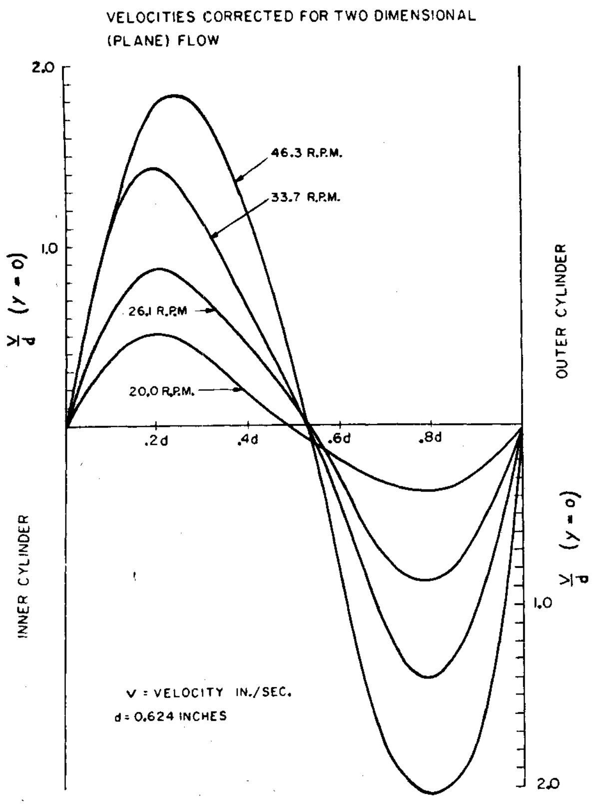


Fig. 7. Velocity distributions as corrected for the effect of curvature of the cylinder walls.

HEAT TRANSFER COEFFICIENT

The next step is to measure the heat transferred across the eddies in the annulus for different rotational speeds of the cylinder and consequently for different speeds of the eddies. These measurements are done in a system which is dynamically and geometrically the same as that used for the motion picture study. The velocity components of the fluid tangential to the cylinders do not affect the heat transferred. Thus if the annulus thickness is small compared with other dimensions the problem is nearly that of heat transfer through a two-dimensional eddy. In the system described the inner cylinder is heated by an electric heating coil to some given temperature (140° to 150°F.) and the outer cylinder is maintained at a fixed temperature (about 140°F.) by means of a water jacket. By computing the heat loss, measuring the temperatures of the inner cylinder and the bulk water, and measuring the power input to the heating coil, the heat transfer coefficient of the system may be found (*3*). When the thermal resistance of the cylinders, the end corrections, and the cooling water are taken into account, the heat transfer coefficient for the eddies alone may be obtained. This procedure is carried out for speeds of the inner cylinder as high as twice the critical speed. It is found that the heat transferred increased linearly with the speed of the inner cylinder.

In the determination of the heat transfer coefficient the annulus is 5/8 in. thick, the inner diameter is 4.235 in., and the length is 12 in. It is also noted that the thickness of the annulus is small compared with the other dimensions, and so the secondary motion can be considered as plane motion. The fluid used is Univis J-58 (hydraulic oil), which has the property of a high-viscosity index and a kinematic viscosity of 0.185×10^{-3} sq. ft./sec. for the temperatures considered. With this fluid the critical speed at which eddies form is about 17.5 rev./min. This accounts for the required correction of Figure 8. The fluid temperatures are maintained within ±10° of 145°F.

Figure 9 is the plot of the total heat transferred, $h_t A$, against the rotational speed of the inner cylinder. h_t is the heat transfer coefficient after end effects are eliminated and A is a cylindrical area taken at the mean radius of the annulus. The temperature difference is taken between the inner surface of the inner cylinder and the bulk temperature of the water in the cooling jacket. The lower dotted line represents the $h_t A$ if pure conduction were the only means of heat transfer. The equation

$$h_t A = \frac{20.2(-146.5 + 9.05n)}{-126.3 + 9.04n} \qquad (6)$$

is an approximate equation of the curve for speeds above 17.5 rev./min. The method of obtaining this expression for $h_t A$ will be developed later. The values of $h_t A$ include the effects of the brass cylinders and the cooling water. However, from the equation for speeds above 17.5 rev./min. it is seen that, as the speed n becomes great, h_t approaches the value of 20.2 B.t.u./(hr.)(°F.).

The method of obtaining an equation like (6) is well known, i.e., the "Wilson plot" to find effects of boundaries, etc.

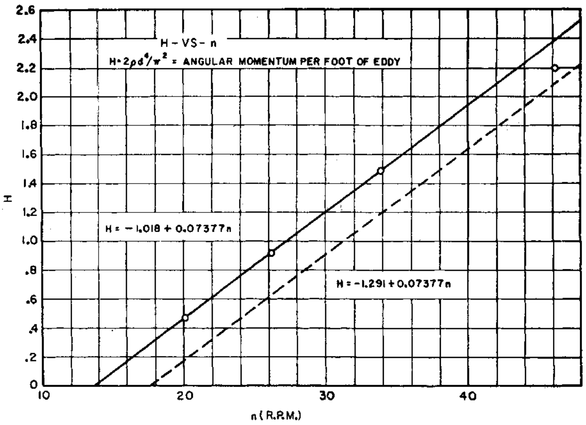


Fig. 8. Angular momentum per foot of eddy vs. rotational speed of inner cylinder.

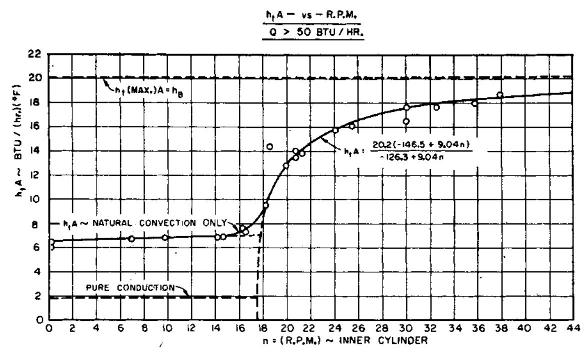


Fig. 9. Total heat transformed vs. rotational speed of the inner cylinder.

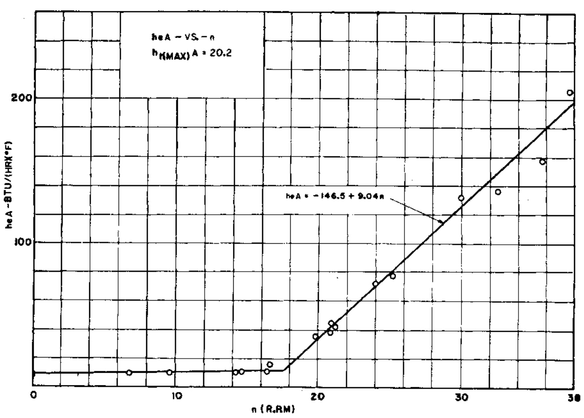


Fig. 10. Heat transfer coefficient for eddies only vs. rotational speed of the inner cylinder.

In the present case, however, the method and reasoning are somewhat different from usual and should be presented. If it is assumed that the thermal resistance of the eddies approaches zero at very high speeds, $h_t\ A_{(max)} = 20.2$ is the heat transfer coefficient of the cylinders and cooling water alone. Defining $h_e A$ as the special heat transfer coefficient of the eddies only ($h_e A = \Delta Q/\Delta T$) gives

$$\frac{1}{h_e A} = \frac{1}{h_t A} - \frac{1}{h_t A_{(max)}} \qquad (7)$$

and $h_e A$ can now be plotted against the revolutions per minute. Figure 10 shows the values of $h_e A$ in terms of the cylinder speed. The equation

$$h_e A = -146.5 + 9.04n \qquad (8)$$

is obtained from Equations (6) and (7).

The method of obtaining the form of Equation (6) can now be shown. If it is assumed that h_e is a linear function of the speed of the inner cylinder n, then it follows that

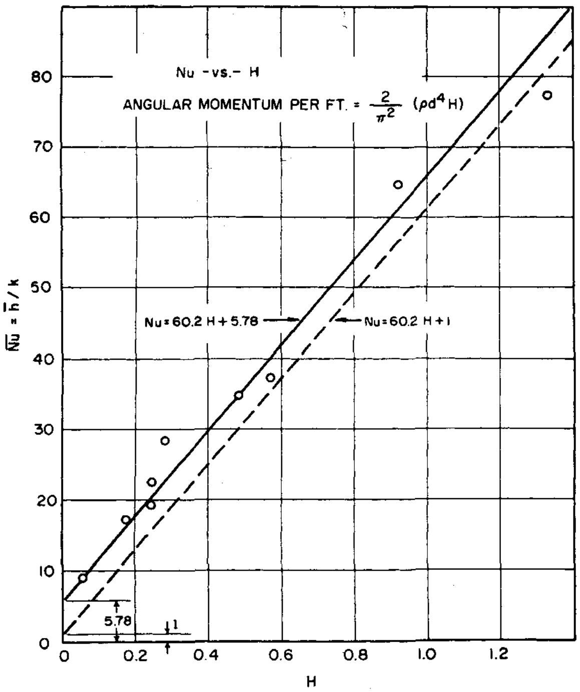


Fig. 11. Heat transfer rate per foot of eddy vs. angular momentum per foot of eddy.

$$h_e A = (a + bn)A \tag{9}$$

$$\frac{1}{h_t A} = \frac{1}{h_t A_{(max)}} + \frac{1}{A(a + bn)} \tag{10}$$

$$h_t = \frac{(a + bn)h_{t(max)}}{a + bn + h_{t(max)}} \tag{11}$$

where a, b, and $h_{t(max)}$ are still undetermined constants. Three experimental points taken from Figure 9 are used to determine these constants, and Equation (6) results. This is also the form of Equation (11).

It is noticed that these equations are of the hyperbolic rather than exponential type, which is usual for the Wilson plot. This is a result of assuming the form of the heat transfer coefficient of the eddies alone to be a linear function of the angular velocity. The reason for assuming $h_e A$ as a linear function of n is evident when one considers the manner in which the angular momentum of the eddy varies with the speed.

Dividing Equation (8) by the product of the number of eddies in the annulus and the mean circumference of the annulus gives a special heat transfer coefficient, $\bar{h}$, the heat transferred per eddy per unit length per degree Fahrenheit, as a function of the revolutions per minute of the inner cylinder.

RELATION BETWEEN ANGULAR MOMENTUM AND HEAT TRANSFER

As a final result one can eliminate the speed and show the relationship between the angular momentum and the heat transfer rate per foot of eddy as in Figure 11. In this plot $\bar{N}_u$ is a special Nusselt number, defined by

$$\bar{N}_u = \frac{\bar{h}}{k} \tag{12}$$

The lower dotted curve in Figure 11 might be expected if there were no natural convection. It would also be the expected lower limit curve for all data. At speeds below critical there is no secondary motion of the eddy type and the special Nusselt number for pure conduction for the equivalent region of one square eddy should be unity if all irregular motions were absent. Thus the curve must pass through $\bar{N}_u = 1$ at $H = 0$. Natural convection makes this condition impossible. The investigation as presented leaves room for experimental error but the error is believed to be less than 8%; this belief is justified in detail by Clark (*3*).

CONCLUSION

In essence the experimental work described has been directed toward a very special problem. For this case the Nusselt number and the angular momentum of an eddy are shown to be linearly related. This result is expected if the analogy between heat transfer by convection and momentum transfer is considered. If it is agreed that the angular momentum of an eddy is a measure of the rate at which fluid moves from one region to another, then it is to be expected that it is an indirect measure of the convective heat transfer coefficient. It is suggested that the angular momentum of an eddy may well be a method of measuring its capacity to transfer heat. However, a stretch of the imagination indicates other possibilities. If turbulence is considered to be a statistical distribution of eddies, which in turn have an effective angular momentum, then possibly there may be a different approach to the study of heat transfer in turbulent flow. A satisfactory mechanistic model to represent turbulence would be required before an investigation in this direction would be fruitful.

The fact that the angular momentum of an eddy was shown to increase linearly with angular velocity (Reynolds number) suggests a different approach to correlating heat transfer data. A detailed discussion of this point will be given in another paper.

Unfortunately heat transfer data could not be obtained for speeds of the inner cylinder greater than about twice the critical, owing to limitations of the recording system used. It is believed that at higher speeds the angular momentum would still maintain a direct proportionality with the heat transferred but that neither would have a linear relation to the angular velocity. Turbulence would certainly change the form of the data. On the other hand, for speeds entirely within the turbulent range there is reason to expect a new linear relation between speed of the inner cylinder and the heat transferred. Further study is to be made on this point.

NOTATION

A	=	effective heat transfer area
a, b, P	=	constants as defined in text
d	=	$(R_2 - R_1)$ annulus thickness
f	=	function of the coordinate x
h_t	=	total heat transfer coefficient
h_e	=	eddy heat transfer coefficient
$\bar{h}$	=	special heat transfer coefficient
H	=	angular-momentum number
k	=	thermal conductivity
n	=	cylinder speed, rev./min.
$\bar{N}_u$	=	$\bar{h}/k$ special Nusselt number
ΔQ	=	heat transfer rate, B.t.u./hr.
R_1, R_2	=	inner and outer radius of annulus
ΔT	=	temperature difference ($h_e A = Q/T$)
u, v	=	velocity components in x and y direction respectively
x, y	=	coordinates of annular eddy
ω_{cr}	=	critical angular velocity in rad./sec.
ρ	=	mass density
ν	=	kinematic viscosity
Ψ	=	stream function

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